

FN-Wahrnehmung- GeomTaeuschungen-EN-v0.1-2026- 05-17

Research Note — Geometric Illusions

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Geometric Illusions **Tool:** ILLUSIO · **Document type:** Research note
(Template v1.0) **Status:** completed test with documented assessment

1. Thesis

The Cave-Allegory Optics Lab (Modules 01–12) ended with the insight that we see the world through apparatuses that limit us — most recently the diffraction limit of every optical instrument (Module 12). The **Perception Lab** carries this thought further to the visual system itself and tests the intuitive assumption that the eye measures lengths and sizes objectively. This assumption is refuted by the classical geometric illusions: **physically exactly equal lengths and sizes are perceived systematically and reproducibly differently as soon as the context changes**. The visual system is not a ruler but an **inference apparatus** — it interprets stimuli in context and produces the most probable interpretation, not an objective measurement. This module is thus the direct, modern continuation of the cave-allegory core (Module 02): what is in the stimulus, and what is our construction from it?

2. Assumptions

1. The physical stimuli (line lengths, circle sizes) are exactly defined and measurable — here equal by construction.
2. The perceptual effects are documented in the psychophysical literature (Müller-Lyer 1889, Ponzo 1911, Ebbinghaus/Titchener).
3. The illusion-strength models used here are **phenomenological** — calibrated to published data, not derived from first principles. This is made transparent.
4. Average adult observer with normal vision; individual and cultural differences are averaged.
5. Static black-and-white line figures; colour and motion effects are separate modules (L2, L4).

3. Mathematical model

M1 — “the eye measures objectively”: Perceived length = physical length. The visual system is a measuring instrument; the ratio perceived/physical is always 1.

M2 — Müller-Lyer: Two equal shafts, one with outward, one with inward wings (arrowheads). The illusion strength depends on the wing angle α and the relative wing length; phenomenological model:

$$S_{\text{ML}}(\alpha) = 0.55 \cdot \sin \alpha \cdot (1 - 0.3 \sin \alpha) \cdot \frac{\ell_{\text{wing}}}{0.3}$$

The outward shaft appears longer, the inward shaft shorter.

M3 — Ponzo: Two equal horizontal bars between converging lines (perspective context). The upper one (nearer the convergence point) appears longer:

$$S_{\text{Ponzo}}(\beta) = 0.5 \cdot \sin \beta \cdot p$$

with convergence angle β and vertical position p .

M4 — Ebbinghaus: A central circle surrounded by circles with radius ratio r (surround/centre). Large $r \rightarrow$ central circle appears smaller:

$$S_{\text{Ebb}}(r) = -0.18 \cdot \tanh(\ln r)$$

Symbol	Meaning	Unit
S	relative illusion strength	—
α	wing angle (Müller-Lyer)	°
β	convergence angle (Ponzo)	°
r	radius ratio (Ebbinghaus)	—
p	vertical position	—

4. Parameters

Three scenarios:

Scenario	Setup
A Müller-Lyer	shaft 200 px, wing angle 15°–75°, standard 30°
B Ponzo	bar 150 px, convergence angle 5°–30°, standard 20°
C Ebbinghaus	central radius 40 px, surround ratio 0.3–3.0

Tolerance set in advance: $\epsilon = 5\%$. Rationale: deviations below 5% count as “perceived as equal” (at the psychophysical difference threshold); above it the illusion is clearly measurable.

5. Prediction

Scenario	Expectation
A	physically equal; perceived outward > inward, deviation $> \epsilon$
B	physically equal; top > bottom
C	physically equal; central circle differs by surround

6. Falsification criterion

M1 (“the eye measures objectively”) is refuted if the perceived size ratio of two physically equal lengths/sizes deviates from 1 by more than $\epsilon = 5\%$ — systematically and reproducibly, by context change alone.

7. Competing models

- **K1 — objective measurement (M1):** tested baseline model.
- **K2 — size constancy / depth hypothesis (Gregory):** the visual system interprets the figures as 3D scenes and applies size constancy (explains Ponzo and partly Müller-Lyer).
- **K3 — lateral inhibition / low-pass filter:** neural low-frequency filtering of the contours shifts the perceived endpoints (explains Müller-Lyer without depth).
- **K4 — Bayesian perception:** the visual system estimates the most probable cause of the stimulus given prior knowledge (most general framework, includes K2/K3).
- **K5 — centre contrast (Ebbinghaus):** size is judged relative to the surround.
- **K6 — cultural shaping:** Müller-Lyer strength varies with the visual environment (“carpentered world” hypothesis, Segall et al.).

8. Domain of failure

- **Individual differences:** illusion strength varies between persons; the model gives means.
- **Cultural differences:** Müller-Lyer is stronger in people from “rectangular” environments; the model is calibrated on Western samples.
- **Extreme parameters:** at very small or very large angles/ratios the effect saturates or vanishes; the model is calibrated in the middle range.
- **Active measurement:** if the observer measures with a ruler, the illusion vanishes — it concerns *perception*, not *knowledge*. Notably, the illusion persists even when one knows the lines are equal (cognitive impenetrability).
- **Dynamic/coloured stimuli:** this module treats static line figures; motion (L4) and colour/brightness (L2) are separate modules.

9. Simulation

Numerical evaluation in `taeuschungen_simulation.py`:

1. Verification of the physical equality of all stimuli (ratio exactly 1.0).
2. Müller-Lyer: illusion strength for five wing angles; standard configuration 30° .
3. Ponzo: illusion strength for four convergence angles; standard 20° .
4. Ebbinghaus: misjudgement of the central circle for five surround ratios; two classical contexts.
5. Comparison of perceived vs. physical ratio against ϵ .

Results in `taeuschungen_results.json`. Plot as `taeuschungen_szenen.svg` (four subplots: Müller-Lyer figure, strength vs. wing angle, Ponzo figure, Ebbinghaus vs. context).

10. Result

10.1 A Müller-Lyer

Quantity	Value
Shaft length (both)	200 px
physical ratio	1.000
illusion strength (30° wings)	0.234
perceived outward	223.4 px
perceived inward	176.6 px
perceived ratio	1.265
Assessment M1	refuted

10.2 B Ponzo

Quantity	Value
Bar length (both)	150 px
physical ratio	1.000
illusion strength (20° convergence)	0.120
perceived top	168.0 px
perceived bottom	150.0 px
perceived ratio	1.120
Assessment M1	refuted

10.3 C Ebbinghaus

Quantity	Value
Central radius (both)	40 px
physical ratio	1.000
perceived (small surround)	45.2 px
perceived (large surround)	34.8 px
perceived ratio	1.300
Assessment M1	refuted

Plot: taeuschungen_szenen.svg.

11. Assessment

M1 (“the eye measures objectively”) is refuted in all three scenarios. In every case the physical ratio is exactly 1.000 — the lengths or circles are identical by construction, and the simulation verifies this. Yet perception deviates systematically and far beyond the tolerance ($\epsilon = 5\%$).

Scenario A (Müller-Lyer) is the most famous geometric illusion. Two exactly equal lines appear different in length merely because wings (arrowheads) point in different directions at their ends: outward wings make the shaft appear longer (223 px instead of 200), inward wings shorter (177 px) — a perceived ratio of 1.265, over 26% difference at zero physical difference. The illusion strength depends on the wing angle. The explanation remains disputed: Gregory interprets the wings as depth cues (an inward figure like a protruding building corner, an outward figure like a receding room corner), the visual system applies size constancy. Alternatively, low-pass filter models explain the shift of the perceived endpoints purely neurally. Notably, the “carpentered world” hypothesis (Segall, Campbell, Herskovits 1966) found that people from environments with many right angles experience the illusion more strongly — an indication that perception is **learned** and culturally shaped.

Scenario B (Ponzo) shows the same mechanism more explicitly. Two equal bars lie between converging lines — like two sleepers on perspectively converging railway tracks. The upper bar, nearer the convergence point, appears ~12% longer. Here the depth hypothesis is immediately plausible: the converging lines signal spatial depth, the upper bar appears “further away”; because it is nevertheless imaged at the same size, the visual system concludes it must be “actually larger” — the size constancy that is correct in the real world (distant things image smaller) misleads in the 2D image.

Scenario C (Ebbinghaus) shows that size is judged **relatively**. A central circle surrounded by small circles appears larger than the same circle surrounded by large circles — a perceived ratio of 1.30 for identical central circles. The visual system has no absolute size scale; it judges relative to the surround. Interesting is the finding that the Ebbinghaus illusion strongly affects conscious size perception but the *grasping movement* less (the hand opens to the correct physical size) — an indication of separate processing pathways for “seeing for recognition” and “seeing for action” (Milner & Goodale).

The central methodological lesson — and the reason this strand has a different character from the optics: here the *physical measurement is trivial* (the lines are equal, done), and the entire phenomenon lies in *perception*. While the optics modules tested a physical law against an idealisation, this module tests perception against physical reality — and finds a systematic, lawful deviation. This shifts the question: it is no longer about *what the world does* but *what our apparatus makes of the world*. This is exactly the question of the cave allegory (Module 02), and the whole perception strand is its systematic execution. Notably, and particularly devastating for M1: the illusions persist **even when one knows the lengths are equal** — perception is sealed off from better knowledge (cognitively impenetrable). One cannot “think one’s way out”.

Connection to previous modules: - **Module 02 Cave Allegory:** the direct predecessor — perception as mediated, never immediate. L1 makes this quantitative. - **Module 09 Colour Mixing:** there *colour* was the construction; here it is *length and size*. The same basic insight (perception $\neq$ physical stimulus) on a new dimension. - **Module 12 Instruments:** ended at the fundamental limit of seeing through apparatuses; L1 shows that the “apparatus eye+brain” itself constructs systematically.

12. Next test

Module L2 Brightness and Colour Contrast as the next step: simultaneous contrast, Mach bands, Adelson checker-shadow — same grey value, different perceived brightness; connects directly to Module 09 (lateral inhibition as neural mechanism). Or **Module L5 bistable perception** (Necker cube, ambiguous figures): one stimulus, two interpretations — the strongest form of falsifying “one image = one perception”.

Possible extension Module L1 v0.2: further illusions (Hering, Zöllner, Poggendorff, café wall); quantitative comparison of the explanatory models (depth hypothesis vs. filter model); the cultural dimension (carpentered world); the perception/action dissociation (Ebbinghaus grasping).

Appendix A — Cross-references

- Practice explanation: PRAXIS-GeomTaeuschungen-EN.md (follows)

- Companion paper: BEGLEITPAPIER-GeomTaeuschungen-EN.md (follows)
- Tool: ILLUSIO.html (follows) — *Lat. illusio, the illusion; root of “illusion”*
- Code: taeuschungen_simulation.py
- Plot: taeuschungen_szenen.svg
- Result data: taeuschungen_results.json
- Predecessor modules: Module 02 Cave Allegory, Module 09 Colour Mixing, Module 12 Instruments
- Follow-up modules (planned): L2 Brightness contrast, L3 Afterimages, L4 Motion, L5 Bistability, L6 Gestalt

Appendix B — External stocktaking

- **Müller-Lyer (1889)** Optical judgement illusions. *Arch. Anat. Physiol.*
- **Ponzo (1911)** — perspective size illusion.
- **Gregory (1997)** *Eye and Brain* — depth hypothesis of illusions.
- **Segall, Campbell & Herskovits (1966)** *The Influence of Culture on Visual Perception* — carpentered world.
- **Milner & Goodale (1995)** *The Visual Brain in Action* — two visual pathways.
- **Gombrich (1960)** *Art and Illusion* — perception and art.

What the workshop note adds: three illusions in a unified falsification format with explicit separation of physical/perceived; the transparent labelling of the phenomenological models; the framing as a continuation of the cave-allegory core; the bridges to modules 02, 09, 12.

Appendix C — Sources

- Müller-Lyer, F. C. (1889). Optical judgement illusions. *Archiv für Anatomie und Physiologie, Physiologische Abteilung* 2:263–270.
- Ponzo, M. (1911). Intorno ad alcune illusioni nel campo delle sensazioni tattili. *Archives Italiennes de Biologie.*
- Gregory, R. L. (1997). *Eye and Brain: The Psychology of Seeing*, 5th ed. Princeton University Press.
- Segall, M. H.; Campbell, D. T.; Herskovits, M. J. (1966). *The Influence of Culture on Visual Perception*. Bobbs-Merrill.
- Milner, A. D.; Goodale, M. A. (1995). *The Visual Brain in Action*. Oxford University Press.