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Research Note — Entanglement and Bell's Inequality

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1. Thesis

Module Q1 (DUALITAS) ended with a gap: the formalism predicts measurement outcomes but tells no story about the in-between. It is natural to fill this gap with an assumption — **the particles carry their answers from the start, we simply do not know them.** That was Einstein's position (EPR 1935): quantum mechanics is incomplete, something is missing.

The tested baseline model M1 reads: *"Local realism. Every particle carries its answers from the start (realism), and what is measured at one place does not depend on what is set at the other (locality). The correlations are explicable by common history."*

This module's claim: M1 is **not merely improbable but experimentally excluded** — regardless of how complicated one makes the hidden variables. Bell (1964) turned a philosophical question into a measurable one.

A preliminary note on the reach of this note: the simulation reproduces predictions. It does not prove the Bell bound — Bell did that analytically — and certainly not that nature violates it. The experiments did that. Whoever infers something about the world from a simulation that has the quantum prediction built in commits a circular argument.

2. Assumptions

1. Two particles are created together and measured separately.
2. On each side one of two settings is chosen, the outcome is \(\pm 1\).
3. A locally realistic model consists of a shared hidden variable and two local response functions.
4. Correlations are measurable as averages over many pairs.
5. Finite samples have noise; the bound holds for expectation values.

3. Mathematical model

M1 — local realism. There exists a hidden variable λ with distribution $\rho(\lambda)$ and response functions $A(a,\lambda), B(b,\lambda) \in \{-1,+1\}$ such that $$E(a,b) = \int A(a,\lambda)\,B(b,\lambda)\,\rho(\lambda)\,d\lambda.$$ **Crucially:** A does not depend on b and B not on a — that is the locality condition.

M2 — the CHSH inequality. For the quantity $$S = E(a_1,b_1) - E(a_1,b_2) + E(a_2,b_1) + E(a_2,b_2)$$ **every** such model satisfies: $$|S| \le 2.$$

M3 — the quantum prediction. For the singlet state $E(a,b) = -\cos(a-b)$. With $a_1=0$, $a_2=\pi/2$, $b_1=\pi/4$, $b_2=3\pi/4$ it follows that $$|S| = 2\sqrt{2} \approx 2.828 \quad\text{(Tsirelson bound)}.$$

M4 — sampling noise. Each correlator has a standard error of about $1/\sqrt{N}$ for N pairs; for S as a sum of four, $\mathrm{SE}(S) \approx 2/\sqrt{N}$.

Symbol	Meaning	Unit
λ	hidden variable	—
$E(a,b)$	correlation	—
S	CHSH quantity	—
SE	standard error	—

4. Parameters

Scenario	Setup
A CHSH	optimal angles; LHV model with N from 1,000 to 1,000,000; plus 300 random angle sets
B angles	correlation over 0° to 180°, quantum prediction against best linear model
C no signalling	Alice fixed at 0°, Bob at six settings, 200,000 pairs each

Tolerance: $\varepsilon = 0$ in A and B, 0.01 in C.

5. Prediction

Scenario	Expectation
A	QM reaches 2.828; LHV stays within a few standard errors of 2
B	agreement at 0°, 90°, 180°; deviation in between
C	Alice's distribution independent of Bob's setting

6. Falsification criterion

M1 is refuted when the measured CHSH quantity exceeds the bound of 2 **significantly** — that is, by a multiple of the standard error. A single value just above 2 does not suffice.

7. Competing models

- **K1 — local realism (M1):** tested baseline; the EPR position.
- **K2 — quantum mechanics:** predicts $2\sqrt{2}$, without hidden variables.
- **K3 — nonlocal hidden variables (Bohm):** saves realism, gives up locality. Not excluded by Bell.
- **K4 — many-worlds:** saves locality, gives up definite outcomes. Likewise not excluded.
- **K5 — superdeterminism:** the measurement settings would not be freely choosable. Not falsifiable and therefore problematic in philosophy of science.
- **K6 — PR boxes:** hypothetical correlations with $|S| = 4$ that would still forbid signalling. Nature does not use them — why is open.

8. Domain of failure

- **The simulation proves nothing about the world.** It computes a locally realistic model and shows that it respects the bound. That nature violates it was shown by experiments.
- **The LHV model is one of many.** Bell's proof is general; here a concrete example is computed, no general proof given.
- **Idealised conditions.** Perfect detectors, no losses, exact angles.
- **Loopholes.** Until 2015 two remained: the detection loophole (not all pairs registered) and the locality loophole (settings could coordinate). Both were closed in 2015 in three independent experiments simultaneously. **Superdeterminism remains open in principle** — but it is not falsifiable.
- **No interpretation.** What is the case instead, once local realism fails, remains open (module W2).
- **Not treated:** GHZ states (refutation without statistics), entanglement measures, applications in quantum cryptography.

9. Simulation

Numerical evaluation in `nexus_simulation.py`:

1. Scenario A: CHSH for the quantum prediction; explicit LHV model for four sample sizes with standard error and significance test; search over 300 random angle sets.
2. Scenario B: correlation over the angular range, quantum prediction against best linear model; determination of the maximum deviation.
3. Scenario C: marginal distributions at six settings of Bob; scatter across all settings.

Results in `nexus_results.json`, plot as `nexus_szenen.svg`.

10. Result

10.1 A — the CHSH bound

Model	$\|S\|$
local realism (bound)	2.000
quantum mechanics	**2.828427**
algebraic maximum	4.000

Excess over the classical bound: **41.42 %**.

The explicit LHV model with significance test:

N	S	SE	σ above 2	significant
1,000	−1.9980	0.063	−0.03	no
10,000	−1.9828	0.020	−0.86	no
100,000	**−2.0080**	0.006	**+1.26**	**no**
1,000,000	−1.9992	0.002	−0.42	no

Best result over 300 random angle sets: 2.0161 — that is **1.14 σ** above the bound, hence noise.

Assessment M1: refuted (by the experiments, not by this calculation).

10.2 B — angular dependence

Angle	QM	best linear model	Difference
0°	−1.0000	−1.0000	**0.0000**
22.5°	−0.9239	−0.7500	0.1739
39.5°	—	—	**0.2105 (max)**
45°	−0.7071	−0.5000	0.2071
90°	0.0000	0.0000	**0.0000**
180°	+1.0000	+1.0000	**0.0000**

Assessment M1: refuted.

10.3 C — no signalling

Alice fixed at 0°:

Bob's setting	P(Alice = +1)	Correlation
0°	0.5006	−1.0000
45°	0.5015	−0.7051
90°	0.4993	+0.0003
180°	0.4985	+1.0000

Scatter of P(Alice = +1): 0.003 — pure sampling noise.

Assessment: delimitation confirmed.

11. Assessment

M1 ("local realism") is refuted — but it is worth stating precisely by what and in what sense.

Scenario A shows the bound and its violation. Quantum mechanics predicts $|S| = 2\sqrt{2} = 2.828$, 41 % above the classical bound. This is no marginal effect but a clear one.

The explicit locally realistic model respects the bound — in all four sample sizes and across 300 random angle sets. **The case N = 100,000 is notable:** there the model yields $|S| = 2.008$, apparently above the bound. This is **no violation** but sampling noise: the standard error is 0.006, the excess amounts to 1.26 σ. The Bell bound holds for **expectation values**, not for finite samples.

This point is more than a footnote. It explains why real Bell experiments must state significances and why the loophole-free tests of 2015 were so demanding — Hensen et al. reported a violation with p = 0.039, which is marginal by today's standards. Whoever sees a value of 2.008 in a simulation and makes a finding of it has mistaken noise for signal.

The scope of Bell's proof is decisive. The calculation here tests **one** locally realistic model. Bell proved analytically that **no** such model can reach $|S| > 2$ — regardless of how λ is chosen, how many dimensions it has, what distribution it follows. The bound follows from locality and realism alone. This is why the experimental violation is so powerful: it excludes not one model but an entire class.

Scenario B contains the historical point. At 0°, 90° and 180° the quantum prediction and the best locally realistic model agree **exactly**. Only in between do they diverge, maximally by 0.21 at 39.5°.

This explains why the discrepancy went unnoticed for so long. Whoever measures only at the "natural" angles — parallel, perpendicular, opposite — sees no difference. **One must choose the right angles to see anything at all.** That is Bell's real achievement: he showed not only that there is a difference but **where** to measure to find it.

Einstein's position was therefore not absurd but the natural one. That particles bring their properties with them and we merely do not know them is the explanation one would give in any classical context — with gloves in two suitcases, say. Bell turned this philosophical position into a **testable claim**. That is the rare case of a foundational question becoming decidable.

Scenario C delimits what the refutation means. Alice's outcome distribution stays practically unchanged across all of Bob's settings (scatter 0.003, pure noise). The **correlation** changes — the **individual outcomes** do not.

It follows that **entanglement permits no signalling.** Alice always sees her half as pure randomness. Only when both bring their data together — which requires classical communication — do the correlations appear.

This is the same structure as the quantum eraser from Q1: what is locally observable never changes, only the subsequent combination reveals the structure. **"Spooky action at a distance" is misleading as an image** if one thinks of transmission. What Bell shows is not that something is transmitted but that the correlations are **not explicable by common history**. That is a statement about explicability, not about action.

What remains open. The refutation excludes local realism — no more. Whoever gives up locality can keep realism (Bohm). Whoever gives up definite outcomes can keep locality (many-worlds). Neither is excluded, and both make **the same predictions**.

The gap from Q1 thus deepens rather than closes. There it emerged: the formalism tells no story about the in-between. Here: **the most natural story — that the answers were already fixed — is excluded.** What is the case instead remains a question of interpretation.

This is the third place where the workshop meets the same pattern. In **M1 (CATENA)** the output did not determine the stimulus uniquely. In the **perception strand** we see not the stimulus but the best interpretation. In **Q1 and Q2**: the theory predicts outcomes and excludes explanations without putting one in their place. Whether that is a deficiency of the theory or a property of the world is the subject of W2.

12. Next test

Module W1 (Measuring, Interpreting, Applying) and **W2 (The Interpretations)** as the conclusion of the workshop. Q1 and Q2 have jointly laid the groundwork: they show precisely where the explanatory gap lies and which fillings are excluded.

Possible extension v0.2: GHZ states (refutation without statistics, with a single measurement outcome); entanglement measures; quantum cryptography as an application; the loopholes in detail; PR boxes and the question why nature stops at \(2\sqrt{2}\).

Appendix A — Cross-references

- Practice explanation: `PRAXIS-Nexus-EN.md` · Companion paper: `BEGLEITPAPIER-Nexus-EN.md`
- Tool: `NEXUS.html`
- Code: `nexus_simulation.py` · Plot: `nexus_szenen.svg` · Data: `nexus_results.json`
- **Connecting modules:** Q1 double slit (the gap), M1 signal chain (non-uniqueness)
- **Planned:** W1/W2 philosophy of science

Appendix B — External stocktaking

- **Einstein, Podolsky & Rosen (1935)** — the incompleteness thesis.
- **Bell (1964)** — the inequality.
- **Clauser, Horne, Shimony & Holt (1969)** — the CHSH form.
- **Aspect et al. (1982)** — the first convincing test.
- **Hensen et al., Giustina et al., Shalm et al. (2015)** — loophole-free tests.
- **Nobel Prize 2022** to Aspect, Clauser and Zeilinger.

What the workshop note adds: the explicit significance treatment showing why an LHV run just above 2 is no finding; the point that agreement at the boundary angles explains the historical delay; the consistent distinction between “explanation excluded” and “action transmitted”; the framing as the third instance of the same workshop pattern.

Appendix C — Sources

- Einstein, A.; Podolsky, B.; Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review* 47:777–780.
- Bell, J. S. (1964). On the Einstein Podolsky Rosen paradox. *Physics* 1:195–200.
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- Aspect, A.; Dalibard, J.; Roger, G. (1982). Experimental test of Bell’s inequalities using time-varying analyzers. *Physical Review Letters* 49:1804–1807.
- Hensen, B. et al. (2015). Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. *Nature* 526:682–686.

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