FN-Optiklabor-Farbmischung-EN-v0.1-2026-05-17

Research Note — Colour Mixing

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1. Thesis

Module 08 (Stage Lighting) showed additive colour mixing on walls. Module 09 takes the fundamental step back and asks: **What is colour, actually?** The intuitive school-physics assumption “every colour corresponds to one wavelength of light” is tested against three findings and turns out to be fundamentally wrong: **colour is not a physical property of light but a perceptual response of three cone types in the eye.** The same colour can be produced by infinitely many different spectra (metamerism), mixed colours often have no corresponding spectral wavelength at all, and the mixing logic (additive vs. subtractive) depends on the physical mechanism, not on “the colour” itself.

2. Assumptions

1. Linear optics: spectra superpose additively on emission.
2. Lambert-Beer for absorption: transmission spectra multiply.
3. CIE 1931 standard observer (2° field) as model of colour perception.
4. Trichromatic vision: three cone types (S, M, L) approximated by CIE colour matching functions $\bar{x}, \bar{y}, \bar{z}$.
5. sRGB colour space (D65 white point) for display.
6. Idealised LED emission spectra as Gaussian bells; CMY filters as inverted Gaussian absorption bands.

3. Mathematical model

M1 — “colour = wavelength”: Every colour corresponds to exactly one wavelength λ. One colour ↔ one wavelength (spectral colour).

M2 — additive mixing + tristimulus: Emitted spectra $S_k(\lambda)$ sum up; colour perception is the tristimulus

$$X = \int S(\lambda)\,\bar{x}(\lambda)\,d\lambda,\quad Y = \int S(\lambda)\,\bar{y}(\lambda)\,d\lambda,\quad Z = \int S(\lambda)\,\bar{z}(\lambda)\,d\lambda.$$

M3 — metamerism: Two spectra $S_1(\lambda) \neq S_2(\lambda)$ with identical tristimulus $(X,Y,Z)_1 = (X,Y,Z)_2$ produce identical colour perception. The mapping spectrum → colour is not injective (infinity-to-one).

M4 — subtractive mixing: Filter transmissions $T_k(\lambda)$ multiply:

$$ S_{\text{out}}(\lambda) = S_{\text{in}}(\lambda)\prod_k T_k(\lambda). $$

Symbol	Meaning	Unit
λ	wavelength	nm
$S(\lambda)$	spectral radiance	rel.
$\bar{x},\bar{y},\bar{z}$	CIE colour matching functions	—
X,Y,Z	tristimulus values	rel.
$T(\lambda)$	filter transmission	0..1
x,y	CIE chromaticity coordinates	—

4. Parameters

Three scenarios:

Scenario	Setup
A additive	LED-R (630nm), LED-G (530nm), LED-B (465nm); mixtures R+G and R+G+B
B metamerism	Spectral yellow (580nm, narrow) vs. optimised R+G mixture
C subtractive	Equal-energy white through cyan, magenta, yellow filters

Tolerance set in advance: $\varepsilon = 5\%$ chromaticity distance (Euclidean in CIE xy). Rationale: 5% in xy roughly corresponds to the discrimination threshold (MacAdam ellipses are smaller, but 5% is a conservative upper bound for "clearly different colour").

5. Prediction

Scenario	Expectation
A	R+G appears yellow but has no energy at 580nm → M1 refuted
B	Spectral yellow and R+G mixture have chromaticity distance $< \varepsilon$ → M1 fundamentally refuted
C	C+M+Y absorbs → dark, opposite to additive R+G+B=white → M1 refuted

6. Falsification criterion

M1 ("colour = wavelength") is refuted if **any** of the following holds: - A mixed colour appears whose tristimulus corresponds to no single spectral wavelength (energy at the "expected" wavelength $< 1\%$).

- Two different spectra produce chromaticity distance $< \varepsilon$ (metamerism). - Additive and subtractive mixing of the same primaries lead to opposite results.

7. Competing models

- **K1 — “colour = wavelength” (M1):** tested baseline model (naive Newton).
- **K2 — trichromatic theory (Young-Helmholtz):** three cone types — M2/M3.
- **K3 — opponent-process theory (Hering):** red-green, blue-yellow, light-dark as opponent pairs; complements trichromacy at the neural level.
- **K4 — CIE 1931 standard observer:** quantitative model — used here.
- **K5 — CIECAM02 / modern appearance models:** account for adaptation, surround — advanced.
- **K6 — spectral rendering:** full spectra instead of RGB — needed for metamerism-accurate simulation.

8. Domain of failure

- **Very dark / very bright stimuli:** CIE 1931 is for photopic (day) vision; at twilight (mesopic) and night (scotopic) sensitivity shifts (Purkinje effect).
- **Very small / large fields:** CIE 1931 applies to 2°; for 10° there is CIE 1964.
- **Colour blindness:** dichromats have only two cone types — own colorimetry.
- **Fluorescence:** substances re-emitting light at a different wavelength break the simple filter multiplication.
- **Metameric failure under different light sources:** two colours matching under daylight may differ under artificial light (well-known problem in textiles).

9. Simulation

Numerical evaluation in `farbmischung_simulation.py`:

1. CIE 1931 colour matching functions $\bar{x}, \bar{y}, \bar{z}$ as analytic Gaussian sums (Wyman-Sloan-Shirley 2013 approximation).
2. Spectrum → XYZ by numerical integration (trapezoidal over 380–780 nm in 5 nm steps).
3. XYZ → sRGB via standard D65 matrix plus gamma correction.
4. Scenario A: LED RGB spectra as Gaussian bells, additive summation.
5. Scenario B: optimisation of R+G mixing ratio to minimise chromaticity distance to spectral yellow (60×60 grid).
6. Scenario C: CMY filters as inverted Gaussian bands, multiplicative transmission.

Results in `farbmischung_results.json`. Plot as `farbmischung_szenen.svg` (three subplots).

10. Result

10.1 A additive RGB mixing

Quantity	Value
R+G appears as	yellow (sRGB ≈ [0.81, 1.0, 0.0])
Energy at 580nm in R+G mixture	0.048 (practically zero)
R+G+B appears as	white (sRGB ≈ [0.76, 1.0, 0.98])
Assessment M1	**refuted**

10.2 B metamerism

Quantity	Value
Spectral yellow (580nm) chromaticity	xy = (0.512, 0.488)
R+G mixture chromaticity	xy = (0.497, 0.470)
Chromaticity distance	0.023 ($< \varepsilon = 0.05$)
Mixing ratio α:β (R:G)	2.16 : 0.94
Assessment M1	**refuted (fundamental)**

10.3 C subtractive CMY mixing

Quantity	Value
after cyan	cyan (sRGB ≈ [0.0, 0.88, 1.0])
after cyan·magenta	blue (sRGB ≈ [0.5, 0.25, 1.0])
after cyan·magenta·yellow	remaining energy only 32% (dark)
Assessment M1	**refuted**

Plot: `farbmischung_szenen.svg`.

11. Assessment

M1 (“colour = wavelength”) is refuted in all three scenarios — and scenario B provides the most fundamental refutation: **metamerism**.

Scenario A shows the most everyday counterexample: every computer monitor produces “yellow” by mixing red and green subpixels — there is **no yellow light** in the physical sense on the screen, only red and green that the eye interprets as yellow. The R+G mixture has practically no energy at 580 nm (spectral yellow). Anyone believing “colour = wavelength” would have to claim the monitor shows no real yellow — which is absurd, because it looks just as yellow as a sodium vapour lamp.

Scenario B is the core: **metamerism**. Spectral yellow (a narrow line at 580 nm) and the R+G mixture (two separate lines) have a chromaticity distance of only 0.023 — they look practically identical. This is only possible because the eye reduces the incoming spectrum to **three numbers** (the responses of the three cone types). Different spectra producing the same three numbers are indistinguishable. The mapping spectrum → colour is infinity-to-one. **Colour is therefore not in the light but a construction of the visual system.**

Scenario C shows that even the mixing rules depend on the mechanism: additive (light sources add spectra) gives R+G+B = white; subtractive (filters/pigments multiply transmissions) gives C+M+Y = dark. The same “colours” behave oppositely depending on whether light is emitted or absorbed. This explains the schoolroom confusion “Why do mixed watercolours give brown/black but mixed lights give white?” — they are two different physical processes.

Connection to previous modules: - **Module 08 Stage Lighting** showed additive mixing on walls (RGB spotlights); Module 09 explains *why* this works (tristimulus) and distinguishes it from subtractive mixing. - **Module 05 Lenses (chromatic aberration)** showed that $n(\lambda)$ is wavelength-dependent; Module 09 shows that the *perception* of these wavelengths collapses through three cone types. - **Module 03 Single Slit / Module 07 Light Sources:** the physical spectrum is objective; colour is subjective-trichromatic.

12. Next test

Module 07 Light Sources as next step: what do real sources emit (incandescent as black-body continuum, LED as narrow bands, fluorescent as lines + phosphor continuum, laser as single line)? How does this relate to colour temperature and colour rendering index (CRI)? Requires Module 09 (tristimulus).

Possible extension Module 09 v0.2: opponent-process theory (Hering), afterimages, colour constancy, MacAdam ellipses, CIELAB colour difference ΔE.

Appendix A — Cross-references

- Practice explanation: `PRAXIS-Farbmischung-EN.md` (follows)
- Companion paper: `BEGLEITPAPIER-Farbmischung-EN.md` (follows)
- Tool: `METAMERIA.html` (follows) — named after the core effect metamerism
- Code: `farbmischung_simulation.py`
- Plot: `farbmischung_szenen.svg`
- Result data: `farbmischung_results.json`
- Predecessor modules: Module 08 Stage Lighting (additive mixing), Module 05 Lenses (chromatic)
- Related modules: Module 07 Light Sources (forthcoming), Module 03 Single Slit (spectrum)

Appendix B — External stocktaking

- **Wyszecki & Stiles (2000)** *Color Science*, 2nd ed. — standard reference of colorimetry.
- **Fairchild (2013)** *Color Appearance Models*, 3rd ed. — modern appearance models.
- **CIE 015:2018** *Colorimetry* — official standard.
- **Wyman, Sloan, Shirley (2013)** *Simple Analytic Approximations to the CIE XYZ Color Matching Functions* (JCGT) — the CMF approximation used here.
- **Hunt & Pointer (2011)** *Measuring Colour*, 4th ed.
- **PhET "Color Vision"** — didactic simulation, without falsification format.

What the workshop note adds: three contrasting scenarios in falsification format; explicit demonstration of the non-injectivity spectrum→colour (metamerism); real CIE CMFs in the numerics; bridges to modules 08, 05, 07.

Appendix C — Sources

- Wyszecki, G.; Stiles, W. S. (2000). *Color Science*, 2nd ed. Wiley.
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- Young, T. (1802). On the Theory of Light and Colours. *Phil. Trans.* 92:12–48.

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